It was a great honor to be asked to write the foreword to the 2011 *Math Survey*. However, I have never had the experience of writing a foreword to a magazine whose contents I have not read! I have written introductions to books that I have written or translated; the job then is pretty easy: I write something about what is actually in the book. But in this case, I only have the past—the old issues of *Math Survey* that I contributed to back in the day, and last year's excellent and highly professional work—and no idea about the present contents of the magazine.

So instead, I will write about the future. I will use this as an opportunity to write sort of a mathematical graduation speech. I will call it *The Three Epigraphs*.

1. GET LOST

In 1998, when I wrote the first edition of *The Art and Craft of Problem Solving* (whose introduction specifically mentions Stuyvesant!), I included an epigraph that was a quote from *Jaguars Ate My Flesh*, a collection of humorous travel essays by Tim Cahill. The quote was:

> The explorer is the person who is lost.

The meaning, as we mathematicians like to say, is "clear." You cannot accomplish anything meaningful without making mistakes. In fact, even if you don't accomplish much that is meaningful, you need to make mistakes. This is a sometimes counterintuitive idea for many Stuyvesant students, trained and rewarded over many years *not* to make mistakes.

One of the hardest things to do, especially for us bright high achievers, is to get used to being not just stupid, but bumblingly stupid. All mathematicians spend most—usually way above 90%—of their waking hours feeling confused at best. That is not a bad feeling, mind you, although it is less good than the rare moments of insight. After all, mathematicians are explorers, and explorers are lost. It's an existential condition of being an explorer.

So, learn to enjoy the sensation of being lost. As you encounter mathematical problems—and here I distinguish *problems*, questions that you do not, at the outset, know how to approach, from *exercises*, which are the things at the end of the chapter that you do for homework—you have no choice but to go on wild goose chases, most of which lead nowhere. At least nowhere relevant to the solution of the problem at hand.

But that's OK. Rarely does it work like this:

- A problem is posed.
- A smart mathematician thinks very hard.
- She solves it!

More often, what happens is:

- A problem is posed.
- A smart mathematician thinks very hard.
- She gets nowhere.
- Later, sometimes much later, she realizes that "nowhere" actually is something that a colleague always wanted to know.
- A new problem is solved!

2. GET OBSESSED

When I wrote the second edition of *The Art and Craft of Problem Solving* in 2006, I added a new chapter, lots of new problems, and a new epigraph. This was a quote from *A Cold Case*, by Philip Gourevitch, a true account of an NYPD murder investigation.

> When detectives speak of the moment that a crime becomes theirs to investigate, they speak of "catching a case," and once caught, a case is like a cold: it clouds and consumes the catcher's mind until, like a fever, it breaks; or, if it remains unsolved, it is passed on like a contagion, from one detective to another, without ever entirely releasing its hold on those who catch it along the way.

This quote really captures what makes mathematical work possible. Being lost, the natural state, is best fueled by obsession. This is not unique to mathematics. Indeed, all creative arts—and math is an art, as well as a science—require not just hard work, but full heart-and-soul dedication. You have probably heard sayings like, "A true writer is someone who *has* to write." The same is true of mathematics. The problems that get solved are the problems that we cannot help but ponder.

That doesn't mean that you should ignore your other courses, although I am not saying that this is so bad. It is possible to be obsessed and even do a little bit of multitasking. Recall that I said earlier that mathematicians spend most of their *waking* hours feeling stupid. A good trick that I have often employed is to fool myself that I have solved a problem, just before drifting off to sleep. This allows me to sleep, and then when I wake, I realize the problem was not solved. But more often than not, a good night's sleep gives me essential rest, and new insights.

It is possible, indeed, essential, to turn a problem off, or at least move it to your intellectual backburners. This way you can work on other things, including other problems, other courses, interacting with people, shopping, etc. But the really good problems always stick around, rattling about in your unconscious, sometimes for years.

The key word is "years." Most student achievement, even, regrettably, at Stuyvesant, depends on speed work, correctly answering questions in a

few minutes or seconds. Even prestigious math competitions like the International Mathematical Olympiad depend on relatively quick work (three problems in four hours). But the "real" world of mathematical problems does not have a timer. We need to train ourselves to adjust to a time scale of contemplation where spending a (mostly fruitless) day on a problem is commonplace, and where months, if not years, of slow and erratic progress are the norm.

So, "catch" some problems, and enjoy the fever!

3. GET TOGETHER

I haven't written a third edition yet, but when I do, I will include the following quote, which is not from a book. Instead, it is something that Ravi Vakil said a few years ago. Ravi is a professor at Stanford, and is one of the greatest young mathematicians of the day. He also wrote a terrific book, *A Mathematical Mosaic*, which I highly recommend. What Ravi said was:

> You know, mathematics is the most social of all the sciences.

This is an important observation, and again, a little counterintuitive. There is a common stereotype of the mathematician in the attic, toiling away, emerging only when he or she solves a famous problem. This was almost the case when Andrew Wiles solved Fermat's Last Theorem a few years ago.

But Wiles' work habits are unusual. Most mathematicians talk to each other a lot. Theorems are written up in journals (such as this one!) but the primary and most efficient means of transmission is informal talks, impromptu blackboard arguments, emails, Skype chats, etc.

Why is math so social? It's not because mathematicians are social. Many of us are shy and awkward, and some of us are even autistic. But paradoxically, this makes it easier to communicate, at least about math. We simply don't worry about trivialities like how someone dresses, their command of English, their sexual orientation, etc. Mathematics has been a famously tolerant discipline, often way ahead of society at large. It is understood that when two mathematicians meet, they have something important in common: a shared obsession, most likely about a particular problem, but if not, a shared capacity to obsess about problems. Not infrequently, two mathematicians from different countries may have more in common with each other than either have with non-mathematicians from their own countries. For we mathematicians belong to our own tribe, a tribe of happy, lost, obsessive explorers.

And also, we need to be social, in order to solve problems. Mathematics is so big, so complex, so open-ended, and human brains are so puny and

small, that we need to cooperate. Internet cooperatives like the Wikipedia are helpful, but there is nothing like a colleague who can mention offhandedly that the problem that you are struggling with reminds her of one that she heard at a conference in Korea on a completely unrelated topic, and that someone in Estonia unsuccessfully worked on yet another related problem and . . . before you know it, the three of you have solved a new problem (not the one you started with)!

So start collaborating. Make sure that you tell all your mathematical friends about all the things that you are working on, especially the things that you are having trouble with. Don't worry about competition or idea-stealing. Mathematics is a giant—indeed, universe-sized—commons, where everyone can and should plant crops; sharing, creating, struggling, and sharing some more.

And don't forget to have fun!

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